Discrete Mathematics
for
Neophytes

(and for Beginners)

Number Theory, Probability, Algorithms, and Other Stuff

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Preface

This book was begun around January 1991, as a response to a need I perceived in the classroom. Students who have not had calculus but intend to pursue the mathematical sciences are not ready for a formalistic proof-laden treatment of the subject. These students are the primary target of the book. At the same time more advanced students are often woefully ignorant of discrete mathematics, and it is hoped that they too might find the book worthwhile. By writing a problem-solving book with plenty of examples, I have found some material is new even to distinguished mathematics professors. The book has been used in the classroom and is oriented mainly to students at the freshman or sophomore level at a typical state university.

Discrete mathematics is such a general area that two authors could write two fairly large texts with few topics in common. Since this is meant to be an introductory text, I chose to emphasize those traditional areas that are discrete: that is I chose to include a fair amount of material from both number theory and probability. Secondly, I chose to emphasize computer science. Hence I introduce algorithms followed by a section on recursion. Graphs are introduced very early in the book because they are used for representation purposes in probability as well as in one of the proofs in the number theory (the proof of Euler's generalization of Fermat's Little Theorem). Having introduced graphs, there is a section devoted to graph searching algorithms and another section devoted to finding shortest paths in graphs. There is a mix of other topics, some quite small, because I think they might be useful to the students at which this book is aimed. In general, the book requires algebra skills but pre-calculus, namely logarithms, show up in only one proof. A few problems are oriented towards computer programming, but there are not enough such problems to make a knowledge of programming critical to studying the book.

I have tried to make the style informal. Students tend to like the informal style more than do reviewers. Informality has a pedagogical advantage. It enables one to give the student information that is nearly impossible to pass along in a formal theorem-proof format. Students also enjoy a humorous approach more than do reviewers. Nonetheless, while keeping the style relaxed and humorous, I try not to waste any time by actually telling jokes. A humorous approach can have the advantage of making the student concentrate on the mathematics rather than the application. For example, one of the most important examples is titled *The Probability of Having Dreaded College Administrator Disease*. The example is quite important in that it demonstrates that interpreting say drug tests can be far trickier than most people would imagine. If however the example uses, say, Disease X, then some students invariably want to know if Disease X, really is Aids or something else. If the example is done with drug testing, then some students will want to know if the given probabilities reflect some actual test.

It is odd that so many people think that informality implies a lack of rigor. This is simply not true, although I will admit that my perception of the need for rigor in a book at this level is less than is currently (and for the last fifty years) the style. However, when a proof is stated formally as a proof, then it is intended to be rigorous. If an informal argument for a result is made, it is not simply some ad hoc argument, but is intended to point the way to a proof. If the statement is made that the technique just used in an example can be extended to a proof of some phenomenon under investigation, then the statement is not made lightly. It means that in this instance, the solution technique can be shown fairly easily to extend to a proof. Throughout the book I employ many

examples that I consider interesting, informative, or of great general pedagogical utility. As often as possible, I try to indicate via footnote where I saw the example. Finally, I would like to thank a student, Robert Traphan, who made some helpful observations.

To the Student

This book is intended for beginners. Some of the most abstract material is in the first couple of sections. If this does not digest well, do not let it deter you! Just continue.

Exercises

Exercises are spread unevenly throughout the book. Answers are given in the back for all exercises.