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Sigma (∑) Notation

The sign ∑, called *sigma*, is a glorified symbol for addition that can be quite useful. It is utilized throughout mathematics, statistics, computer science and all other mathematical disciplines. With the ∑ there is usually an *index* that typically is an i or j. Rather than define the ∑ operation, I am going to try to teach it through a sequence of examples. Also, I will state some useful laws involving ∑.

$$\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$$

$$\sum_{i=1}^{4} x^i = x^1 + x^2 + x^3 + x^4$$

$$\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5$$

$$\sum_{j=-2}^{2} j^2 = (-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2$$

$$\sum_{i=1}^{5} 1 = 1 + 1 + 1 + 1 + 1$$

$$S = \{1,2,4\} \quad \sum_{k \in S} x_k = x_1 + x_2 + x_4$$

$$\sum_{i=1}^{\infty} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots$$

$$\sum_{j=0}^{3} x_{j+1} = x_1 + x_2 + x_3 + x_4$$

$$\sum a \cdot x_i = a \sum x_i$$
$$\sum (y_i + x_i) = \sum y_i + \sum x_i$$
$$\sum_{i=1}^{n} a = n \cdot a$$
$$\sum_{i=m}^{n} a = (n - m + 1)a$$

General Laws

☐ **Exercise 1** Convince yourself that the four laws above are true.

☐ **Exercise 2** Prove that it is **not true** that: $\sum_{i=1}^{n} (x_i \cdot y_j) = \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i$

Three useful formulas that we proved in Section 6 (on induction) can be stated as:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. None of the laws is difficult to prove. However, if you cannot do a proof, convince yourself through examples that all four laws are true. Note that the third law is a special case of the fourth law.

2. Almost any example will work. Try $n = 2$ with $a_1 = 1$, $a_2 = 1$, $b_1 = 2$, $b_2 = 2$.